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RATIONAL EXPECTATIONS AND THE ROLE OF MONETARY POLICY

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1. Introduction

The purpose of this paper is to analyze the role of monetary policy in a model with three major characteristics: (1) prices and quantities are competitively determined by market-clearing relationships – that is, by the solution of a competitive equilibrium system; (2) information is imperfect; and (3) expectations of future variables are formed rationally, in the sense of being optimal predictions based on the available information. The focus of the analysis is on the effects of monetary expansion on prices and outputs.

Part 2 of the paper generates a Phillips-curve-type relation in a framework that builds on the work of Friedman (1968) and Lucas (1973). The source of the Phillips curve is a lack of full current information that prevents individuals from dichotomizing unanticipated price movements into relative and absolute components. Hence, although suppliers and demanders in any market form their expectations about future prices (in other markets) in a rational manner, the implied behavior of output in each market does not separate unanticipated supply and demand shifts into relative and aggregate parts. In this framework changes in money that are not fully perceived as nominal disturbances can lead to movements in output. It also follows here that an increase in the variance of the monetary growth rate (which is one component of the variance of aggregate excess demand) induces individuals to attribute a larger fraction of observed price movements to monetary forces, and thereby leads to a reduced responsiveness of output to a given monetary disturbance. Thus, as in Lucas's model, the magnitude of the Phillips curve slope is inversely related to the variance of the monetary growth rate.

Part 2 of the paper also discusses the determination of the variance of relative

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prices and the variance of future prices about their currently predictable values. One interesting conclusion is that an increase in the variance of aggregate excess demand would lead to an increase in the variance of relative prices.

The third part of the paper follows Sargent and Wallace (1975) by inquiring into the role of monetary policy in this type of rational expectations model. Pure variance of money leads to an increase in the variance of output about its full current information position and to an increased variance of future prices about their currently predictable values. Essentially, an additional amount of monetary noise makes it more difficult for individuals to isolate real shifts, and therefore tends to move output away from full information output. Accordingly, to the extent that direct costs of controlling money are neglected, a zero variance of money would be optimal.

I then consider the implications of feedback effects from observed economic variables to money. When the monetary authority lacks superior information, it turns out that this sort of feedback is irrelevant to the determination of output. Essentially, when individuals know the form of the feedback rule and also perceive the variables to which money is reacting, this type of monetary behavior would be taken into account in the formation of expectations. When the monetary authority possesses superior information there is the potential for beneficial countercyclical policy. However, the provision of the superior information to the public has identical implications for output if the costs of providing this information are neglected.

Finally, I consider the case where the monetary authority has superior information about its own monetary rule. This situation might permit a form of systematic policy deception in the 'short run', when the public does not appreciate the nature of the deception. However, in my model where the policy criterion concerns the gap between actual and full information output, this type of policy deception is not desirable.

2. A rational expectations model with imperfect information

2.1. Setup of the model

The model is an extension to the one developed by Lucas (1973). There is one type of nondurable commodity, denoted by y, that can be viewed as a personal service. With this view, the supply of the commodity corresponds to the supply of factor services, and the demand for the commodity corresponds to the demand for factor services. The commodity is transacted in various markets, indexed by $z = 1, \ldots, n$, that are at physically separated locations. The variety of locations for a single good is intended to serve as a proxy for markets in a variety of goods, since the multilocation context seems easier to formalize. There is assumed to be an instantaneous flow of information within any market, but a lag in the flow of information across markets. Hence, at a given point in time,

there can be a different commodity price across markets, but only a single price within each market.

The model is constructed in a discrete time period framework, where the length of the period signifies the time delay with which information travels across markets. In the present setup market participants possess full information about the relevant economy-wide variables with a one-period lag. During a single period an individual can visit any of the n markets, but it is assumed to be impossible to visit more than one market during a period.[1] Further, it is supposed that there is sufficient information about last period's prices across markets so that current arbitrage insures that all markets offer the same ex ante distribution of price. In this respect the setup resembles the one used by Mortensen (1974).

Aside from lagged knowledge of aggregate variables, a participant in market z also possesses current information that is assumed to be limited to an observation of the current price in that market, $P_t(z)$. The crucial idea is that certain types of local information are received more rapidly than some aspects of global information – such as prices in other markets. It is this differential information structure that allows for a confusion between relative and absolute shifts, and thereby allows for temporary real effects of unpreceived money movements. Of course, the use of a fixed-length information lag and the distinction of only two types of information, local and global, are abstractions made solely for technical convenience.

Aside from the nondurable commodity, the only other good in the economy is fiat money, M.[2] Money is held by individuals because it is the only available store of value. New money enters the economy as transfer payments from the government. These transfers are received by individuals at the start of each period in an amount that is independent of each individual's money holding during the previous period. It is assumed, for simplicity, that the government does not participate directly in the commodity markets.

In order to keep the model analytically manageable, I have constructed the equations in log-linear form. All of the variables used below in these equations are to be interpreted in logarithmic terms.

The supply of the commodity at date t in market z, denoted by $y_t^s(z)$, is assumed to depend on the following set of variables: (1) A systematic supply term, $k_t^s(z)$, that is intended to capture systematic changes in technology, popu-

[1]The present analysis does not deal with optimal search for information across markets. Further, the manner in which aggregate information is transmitted (with a one-period lag) is not explored. Extensions to incorporate optimal search could be very interesting.

[2]Sargent and Wallace (1975) have constructed a model that is similar in some respects, but which also includes a capital market. As Bob Lucas has pointed out to me, the existence of a single, economy-wide capital market implies that the observation of the price (rate of return) on this market conveys important aggregate information. This sort of current aggregate information is very different from the current local information that I assume is available in the present model. I plan to deal at a later time with the different type of information structure that is implied by the existence of an economy-wide capital market.

lation, etc. (2) A term that measures the current price of output in market z, $P_t(z)$, relative to the price that is expected to prevail next period.[3] Because all of next period's markets have the same ex ante distribution of price, it can be assumed that this expectation applies to P_{t+1}, the (geometric, unweighted) average of prices across the markets at date $t+1$. If $I_t(z)$ denotes the information possessed at date t by participants in market z,[4] then $\mathrm{E}P_{t+1}|I_t(z)$ is the relevant price expectation. A positive response of supply to the term $P_t(z)-\mathrm{E}P_{t+1}|I_t(z)$ can be viewed as an effect of speculation over time associated with the intertemporal substitutability of leisure. This type of effect has been discussed in Lucas and Rapping (1970) and Lucas (1972). (3) A wealth variable, measured as $[M_t+\mathrm{E}\Delta M_{t+1}|I_t(z)-\mathrm{E}P_{t+1}|I_t(z)]$,[5] that is assumed to have a positive effect on desired leisure, and hence a negative effect on factor supply and a negative effect on $y_t^s(z)$. The inclusion of the (log of the) aggregate money stock, M_t, reflects a simplifying assumption that the total money possessed by participants in market z, $M_t(z)$, is always the same fraction of the aggregate money stock.[6] The term $\mathrm{E}\Delta M_{t+1}|I_t(z)$, where $\Delta M_{t+1} \equiv M_{t+1}-M_t$, accounts for the expected governmental transfer at the start of the next period. (4) Random terms u_t^s and $\varepsilon_t^s(z)$, where u_t^s is a shift term on aggregate supply, and $\varepsilon_t^s(z)$ is a shift term on relative supply in market z. The sample mean of $\varepsilon_t^s(z)$ across the markets is zero by definition. Other properties of the distributions of these random variables will be discussed below, after the introduction of the demand side of the model.

The specific form of the supply function (in log-linear terms) is

$$y_t^s(z) = k_t^s(z)+\alpha_s[P_t(z)-\mathrm{E}P_{t+1}|I_t(z)] - \beta_s[M_t+\mathrm{E}\Delta M_{t+1}|I_t(z)-\mathrm{E}P_{t+1}|I_t(z)]+u_t^s+\varepsilon_t^s(z), \quad (1)$$

where α_s and β_s are, respectively, the (absolute values of the) relative price and wealth elasticities of current commodity supply. It should be noted that the wealth term in eq. (1) does not hold constant the appropriate measure of wealth when $P_t(z)$ changes. The price deflator for the wealth term should be a (weighted) price index that includes $P_t(z)$ as well as $\mathrm{E}P_{t+1}$. The full effect of $P_t(z)$ on $y_t^s(z)$ includes a positive substitution effect plus a reinforcing effect that derives from the reduction in appropriately measured wealth. However,

[3]The inclusion of expected prices at dates further into the future does not seem to have any important effects in the present model.

[4]The present framework is sufficiently simple so that all participants in a single market at a given point in time have the same information set. In this respect, Lucas (1975) considers a more complicated setup.

[5]It would seem preferable to subtract off the expected desired real money holding at date $t+1$. However, this change would complicate the exposition of the model without affecting the main results. See the discussion at the end of appendix 2.

[6]Lucas (1972) develops a model in which this fraction is a random variable. In my model the random relative disturbance terms, discussed next, serve the same purpose.

this wealth effect would be negligible if the weight of the current period, during which $P_t(z)$ applies, is small relative to the weight of the future period(s), during which EP_{t+1} applies. Subsequently, the important issue concerns the net effect of EP_{t+1} on $y_t^s(z)$, which depends from eq. (1) on the sign of $\beta_s - \alpha_s$. If the current period is viewed as short relative to the length of the future, then the substitution effect, as measured by α_s, would tend to dominate over the wealth effect β_s. It is assumed below that the substitution effect is dominant, so that $\alpha_s > \beta_s$ and the net effect of EP_{t+1} on $y_t^s(z)$ is negative.

The specification of the demand side of the model is parallel to that of the supply side,

$$y_t^d(z) = k_t^d(z) - \alpha_d[P_t(z) - EP_{t+1}|I_t(z)] + \beta_d[M_t + E\Delta M_{t+1}|I_t(z) - EP_{t+1}|I_t(z)] + u_t^d + \varepsilon_t^d(z). \quad (2)$$

Price speculation by demanders implies a negative effect of $[P_t(z) - EP_{t+1}|I_t(z)]$ on $y_t^d(z)$, as measured by the elasticity $-\alpha_d$. Note that demanders in market z are assumed to possess the same information set, $I_t(z)$, as suppliers to this market.[7] The positive effect of wealth on commodity demand is measured by the elasticity β_d. As discussed in the case of supply, it is assumed that the substitution effect of EP_{t+1} is dominant, so that $\alpha_d > \beta_d$ applies. Finally, u_t^d and $\varepsilon_t^d(z)$ are stochastic shift terms that are analogous to those introduced into the supply function.

2.2. *Market-clearing determination of prices and outputs*

Before deriving the market-clearing conditions, it is useful to define the excess demand variables,

$$k_t(z) \equiv k_t^d(z) - k_t^s(z),$$

$$u_t \equiv u_t^d - u_t^s,$$

$$\varepsilon_t(z) \equiv \varepsilon_t^d(z) - \varepsilon_t^s(z).$$

The determination of prices depends solely on excess demand measures, but the determination of output (below) requires also the specification of separate supply and demand functions. I assume in the main text that u_t is generated by a random walk process,

$$u_t = u_{t-1} + v_t,$$

$$v_t \sim N(0, \sigma_v^2),$$

[7] In the present setup an individual supplies and demands commodities simultaneously in the same market. It would be possible to allow each individual to visit two markets in each period, one for supply and one for demand, but the resulting complications in information sets are considerable. Separate concepts of supply and demand can be maintained here if one thinks of the commodity that an individual supplies as not being identical to those he demands (for example, back-scratching services?).

where N refers to the normal distribution. The random variable v_t is serially independent and represents the current period's 'innovation' to real aggregate excess demand. Because u_t has a one-to-one effect on u_{t+1}, the v-innovations are 'permanent' in the sense of determining the most likely position of all future values of u. I consider in appendix 1 the implications of substituting a first-order Markov process, $u_t = \lambda u_{t-1} + v_t$, where $0 \leqq \lambda \leqq 1$. In this formulation, smaller values of λ signify that v_t has more of a transitory and less of a permanent effect on excess demand.

I assume that $\varepsilon_t(z)$ is serially independent and independent of u_t,[8] where

$$\varepsilon_t(z) \sim N(0, \sigma_\varepsilon^2).$$

The ε-shifts are purely transitory, in the sense of lasting only one period. This assumption seems best viewed as reflecting substantial arbitrage possibilities across markets over one period of time, rather than implying that relative shifts in taste, technology, etc., are short-lived.

Finally, it is useful to define the (absolute value of the) price elasticity of excess demand,

$$\alpha \equiv \alpha_s + \alpha_d,$$

and the wealth elasticity,

$$\beta \equiv \beta_s + \beta_d.$$

The earlier assumptions on the dominance of substitution effects imply $\alpha > \beta$.

The price at date t in market z is determined to equate supply and demand in that market.[9] Equating the expressions in eqs. (1) and (2), and using the above definitions, leads to the market-clearing condition for market z,

$$\alpha P_t(z) = (\alpha - \beta)\mathrm{E}P_{t+1}|I_t(z) + \beta[M_t + \mathrm{E}\Delta M_{t+1}|I_t(z)] + k_t(z) + u_t + \varepsilon_t(z). \tag{3}$$

[8]Since the sample mean of $\varepsilon_t(z)$ is zero by definition, the distribution of $\varepsilon_t(z)$ would actually depend on the number of markets. However, this consideration seems unimportant if the number of markets is large, as I am implicitly assuming. It is also possible that certain markets have positive or negative correlations of their relative excess demand shifts with the aggregate excess demand shifts (u_t or the monetary disturbance that is discussed below). In that case there would be different information about the aggregate picture in different markets. The analysis would then have to consider the implications of this differential information on the choice of which market to enter. I do not deal with these possibilities here.

[9]The model does not include any elements of adjustment costs for price changes that would inhibit price flexibility. In this respect see Barro (1972). In particular, there is no consideration of the role of long-term nominal contracting in the present analysis. The role of price stickiness in Keynesian models is discussed in detail in Barro and Grossman (1975, ch. 2). It would be of interest to incorporate price adjustment costs into a rational-expectations-type model.

It is apparent from the form of eq. (3) and the assumed serial independence of $\varepsilon_t(z)$ that the distribution of $P_t(z)$ can be independent of the market index, z, only if $k_t(z)$ is constant across markets. That is, the arbitrage condition that insures that all markets have the same ex ante distribution of price is that the ratio of systematic demand to supply, $k_t(z) \equiv k_t^d(z) - k_t^s(z)$, be the same for all markets.[10] It is then permissible to drop the z subscript from the $k_t(z)$ term in eq. (3).

Eq. (3) indicates that $P_t(z)$ is determined by a set of 'demand-pull' variables that include the money stock plus expected next period's transfer and the sum of systematic and random excess demand terms, $k_t + u_t + \varepsilon_t(z)$. There is also a 'cost-push' term, $EP_{t+1}|I_t(z)$,[11] that has an effect in the direction of the sign of $\alpha - \beta$. Under the assumption of a dominant substitution effect, the impact of EP_{t+1} on $P_t(z)$ is positive.

The key element of the rational expectations approach is that the EP_{t+1} term in eq. (3) is not determined by an ad hoc expectations mechanism from outside of the model, but is instead based on the knowledge – implied by an assumed knowledge of the model – that prices are determined by market-clearing conditions of the form of eq. (3). Hence, current market-clearing prices and (the set of) expectations about future prices are determined through a simultaneous process. In order to implement this approach, it is necessary to complete the specification of the model by describing the processes that generate M_t and k_t. I assume, provisionally, that changes in money are generated by a constant growth rate, g, plus a random term, denoted by m_t. That is,

$$M_t - M_{t-1} \equiv \Delta M_t = g + m_t,$$
$$m_t \sim N(0, \sigma_m^2), \tag{4}$$

where m_t is assumed to be serially independent, as well as uncorrelated with v_t and the array of $\varepsilon_t(z)$. I examine the implications of more complicated money supply processes in a later part of the paper. In order to focus on the short-run, cyclical effects of money, I also abstract in the main text from long-term monetary growth – that is, $g = 0$ is assumed. This abstraction amounts to neglecting the effects of systematic inflation. Appendix 2 deals with the case where $g \neq 0$. When $g = 0$, eq. (4) implies that $E\Delta M_{t+1}|I_t(z) = 0$.

I assume that the k_t process takes the form,

$$k_t = k_0 - \beta\rho t, \tag{5}$$

where $k_0 = 0$ can be assumed subsequently through an appropriate normalization of output units. It turns out, as shown in Appendix 2, that the form for k_t

[10]Alternatively, if some serial dependence in $\varepsilon_t(z)$ had been introduced, then $k_t(z)$ could be such as to just offset the implications of this serial dependence for $P_t(z)$.

[11]The terms on the right side of eq. (3) can be viewed equivalently as negative forces on the current excess demand for money.

in eq. (5) amounts to assuming that the systematic growth rate of output is equal to ρ. Again, it is convenient to abstract in the main text from the effects of long-term growth so that $\rho = 0$ is assumed. Since $k_t = 0$ in this case, the systematic excess demand term no longer appears in the analysis. Appendix 2 deals with the case where $\rho \neq 0$.

The next step is to solve the model in the sense of determining prices and outputs as functions of exogenous variables. The simplest procedure for solving the model involves, first, writing out the (log-linear) form of the solution for $P_t(z)$ in terms of a vector of unknown coefficients on the set of relevant independent variables. Second, the market-clearing condition expressed in eq. (3) is used to determine the values of the unknown coefficients. This solution method has been used before in a parallel context by Lucas (1972, 1973). The procedure is analogous to applying the method of undetermined coefficients to a trial solution in the case of differential or difference equations. In the present situation $P_t(z)$ depends on the following variables (in a log-linear form),[12]

$$P_t(z) = \Pi_1 M_{t-1} + \Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z) + \Pi_5 u_{t-1}, \tag{6}$$

where the Π's are the unknown coefficients. Since M_{t-1} is included in current information sets, previous values of M do not appear in the price solution. Since m_t, v_t and $\varepsilon_t(z)$ are serially independent, past values of these variables do not appear.[13] Since, for a given value of v_t, u_t depends only on u_{t-1}, it follows that values of u prior to $t-1$ do not appear.

If individuals know that prices in each period are determined by eq. (6), then the expected price for next period must be

$$\begin{aligned} \mathrm{E}P_{t+1}|I_t(z) &= \Pi_1 \mathrm{E}M_t|I_t(z) + \Pi_5 \mathrm{E}u_t|I_t(z) \\ &= \Pi_1 \mathrm{E}(M_{t-1} + m_t)|I_t(z) + \Pi_5 \mathrm{E}(u_{t-1} + v_t)|I_t(z), \end{aligned}$$

since the expected values of m_{t+1}, v_{t+1} and $\varepsilon_{t+1}(z)$, conditioned on $I_t(z)$, are all zero. The information set, $I_t(z)$, is assumed to include observations (or sufficient data to infer the values) of M_{t-1} and u_{t-1}. The additional information contributed by an observation of $P_t(z)$[14] amounts, from eq. (6), to an observation

[12] In the case where g and ρ are nonzero, the solution includes the additional terms $\Pi_6 t + \Pi_7$ – that is, a time trend and a constant term. See appendix 2.

[13] If a variable such as v_{t-1} had been entered, it would eventually be determined that its associated Π-coefficient was zero.

[14] I have not included an individual's own value of ΔM_t, which arrives as a government transfer, as an additional element of $I_t(z)$. This exclusion is satisfactory if the relation between individual and aggregate transfers is sufficiently noisy so that the individual transfer provides a negligible increment of information over $P_t(z)$. This assumption need not be inconsistent with my earlier simplifying assumption that $M_t(z)$ was a constant fraction of M_t, since $M_t(z)$ refers to the total money contained in market z. If individual and aggregate M were always proportionately related, then M_t would, itself, become an element of $I_t(z)$, and the principal information gap in the model would disappear.

of the sum $\Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z)$. The key to the formation of price expectations is then the calculation of the two expectations, Em_t and Ev_t, conditioned on the observation of $P_t(z)$. In effect, these two conditional expectations are obtained by running regressions of m_t and v_t, respectively, on the observed sum $\Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z)$. That is,

$$Em_t | I_t(z) = \frac{\theta_1}{\Pi_2}\left[\Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z)\right],$$

where

$$\theta_1 = \frac{(\Pi_2)^2 \sigma_m^2}{(\Pi_2)^2 \sigma_m^2 + (\Pi_3)^2 \sigma_v^2 + (\Pi_4)^2 \sigma_\varepsilon^2},$$

and

$$Ev_t | I_t(z) = \frac{\theta_2}{\Pi_3}\left[\Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z)\right], \tag{7}$$

where

$$\theta_2 = \frac{(\Pi_3)^2 \sigma_v^2}{(\Pi_2)^2 \sigma_m^2 + (\Pi_3)^2 \sigma_v^2 + (\Pi_4)^2 \sigma_\varepsilon^2}.$$

The θ_1-coefficient measures the fraction of the total price variance (of $P_t(z)$ about its best estimate given I_{t-1}) that is produced by (aggregate) money variance m, and the θ_2 coefficient measures the fraction produced by aggregate real variance v. The remaining fraction of price variance, $1-\theta_1-\theta_2$, is attributable to relative real variance ε. The expected price at $t+1$ can then be written as

$$EP_{t+1} | I_t(z) = \Pi_1 M_{t-1} + \left(\frac{\Pi_1 \theta_1}{\Pi_2} + \frac{\Pi_5 \theta_2}{\Pi_3}\right)\left[\Pi_2 m_t + \Pi_3 v_t + \Pi_4 \varepsilon_t(z)\right] + \Pi_5 u_{t-1}. \tag{8}$$

The Π-coefficients must be such that the market-clearing condition, eq. (3), holds as an identity, given eqs. (6) and (8). This identity relation implies five (independent) conditions corresponding to term-by-term coefficient equalities for the variables that appear in eq. (6). The algebra is straightforward and I will limit the discussion here to a presentation and interpretation of the results. The five Π-coefficients can be determined to be

$$\begin{aligned}
\Pi_1 &= 1, \\
\Pi_2 &= (\theta_1 + \theta_2) + (\beta/\alpha)(1 - \theta_1 - \theta_2), \\
\Pi_3 &= \Pi_2/\beta, \\
\Pi_4 &= \Pi_2/\beta, \\
\Pi_5 &= 1/\beta.
\end{aligned} \tag{9}$$

These coefficients imply the price solution

$$P_t(z) = M_{t-1} + [\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2)] \times [m_t + (1/\beta)(v_t + \varepsilon_t(z))] + (1/\beta)u_{t-1}. \tag{10}$$

Before discussing this result, it is convenient to define the variance of the total aggregate disturbance, $\beta m_t + v_t$, as

$$\sigma_A^2 \equiv \beta^2\sigma_m^2 + \sigma_v^2.$$

The θ-coefficients in eq. (10) are then determined, using eqs. (7) and (9), as

$$\theta_1 + \theta_2 = \frac{\beta^2\sigma_m^2 + \sigma_v^2}{\beta^2\sigma_m^2 + \sigma_v^2 + \sigma_\varepsilon^2} = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_\varepsilon^2}. \tag{11}$$

An aggregate 'price index', P_t, can be calculated for future reference as a (geometric, unweighted) average of the prices determined in eq. (10),

$$P_t = M_{t-1} + [\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2)][m_t + (1/\beta)v_t] + (1/\beta)u_{t-1}. \tag{12}$$

The relative disturbances, $\varepsilon_t(z)$, are averaged out in determining P_t.

The main results for $P_t(z)$ in eq. (10) can be interpreted as follows. First, M_{t-1}, which is contained in the information set $I_t(z)$, has a proportional effect on $P_t(z)$ ($\Pi_1 = 1$). On the other hand, m_t, the random part of M_t, is not a part of $I_t(z)$. Since market participants do not have separate observations of $P_t(z)$ and the aggregate price index, P_t, they cannot separate the impact of m_t from the impact of the other excess demand shifts, $v_t + \varepsilon_t(z)$. Hence, m_t and $(1/\beta)(v_t + \varepsilon_t(z))$ enter with a common coefficient in eq. (10). This coefficient is less than one as long as the substitution effect of a change in EP_{t+1} outweighs the income effect ($\alpha > \beta$). Because m_t has a coefficient that generally differs from one, it also turns out that this part of money can have a nonzero effect on output.

The expected future price level follows from eqs. (8) and (9) as

$$EP_{t+1}|I_t(z) = M_{t-1} + (\theta_1 + \theta_2)[m_t + (1/\beta)(v_t + \varepsilon_t(z))] + (1/\beta)u_{t-1}. \tag{13}$$

Note that $\theta_1 + \theta_2$ is the fraction of total excess demand variance that is accounted for by aggregate shifts (m and v). Since M and u are generated by random walk processes, it follows that the m_t and v_t shifts persist into the next period and therefore continue to affect P_{t+1}. On the other hand, the $\varepsilon_t(z)$ shift is transitory and does not affect P_{t+1}. Accordingly, the current excess demand shift, $m_t + (1/\beta)(v_t + \varepsilon_t(z))$, is weighted by $\theta_1 + \theta_2$ in forming EP_{t+1}. This weighting would change if the processes that generated M_t, u_t, and $\varepsilon_t(z)$ were altered. Appendix 2 deals with the case where u_t is no longer a random walk.

The difference between current observed price in market z and the expected future price (in any market) is determined from eqs. (10) and (13) as

$$P_t(z) - EP_{t+1}|I_t(z) = (\beta/\alpha)(1-\theta_1-\theta_2)[m_t+(1/\beta)(v_t+\varepsilon_t(z))]. \quad (14)$$

The weighting term on the current demand shift depends on $1-\theta_1-\theta_2$, which measures the fraction of total excess demand variance that is attributable to relative (hence, in this model, transitory) shifts.

The price solutions from eqs. (13) and (14) can be substituted into either the supply or demand function for commodities (eqs. (1) or (2)) to obtain an expression for output. It is convenient to define here the parameter

$$H \equiv \alpha_s\beta_d - \alpha_d\beta_s.$$

Then, the result for output is [15]

$$\begin{aligned} y_t(z) = {} & (H/\alpha)(1-\theta_1-\theta_2)m_t + (1/\alpha)[\alpha_s-(H/\beta)(\theta_1+\theta_2)][v_t^d+\varepsilon_t^d(z)] \\ & + (1/\alpha)[\alpha_d+(H/\beta)(\theta_1+\theta_2)][v_t^s+\varepsilon_t^s(z)] + (\beta_d/\beta)u_{t-1}^d \\ & + (\beta_s/\beta)u_{t-1}^s. \end{aligned} \quad (15)$$

There are a number of interesting aspects of the output expression. First, (only) the unperceived part of the current money stock, m_t, has an impact on output. The sign of the effect depends on the substitution and wealth elasticities of the commodity supply and demand functions, as measured by the combination $H \equiv \alpha_s\beta_d - \alpha_d\beta_s$. In Lucas's (1973) model, the substitution effect on demand, α_d, and the wealth effect on supply, β_s, were both assumed to be zero. In that case unperceived monetary expansion has, unambiguously, a positive effect on output. More generally, this result follows if the substitution effect on supply, α_s, and the wealth effect on demand, β_d, are the dominant influences.[16] I will treat the case where $H > 0$ as the normal one, although there is nothing in my particular model that suggests that this case would typically arise.[17]

Second, the magnitude of the effect of m_t on $y_t(z)$ – which could be called a (reverse) Phillips curve slope – depends, through the $1-\theta_1-\theta_2$ term, on the relation between the variances of relative and aggregate disturbances. In particular,

$$1-\theta_1-\theta_2 = \frac{\sigma_\varepsilon^2}{\sigma_A^2+\sigma_\varepsilon^2} = \frac{\sigma_\varepsilon^2}{\beta^2\sigma_m^2+\sigma_v^2+\sigma_\varepsilon^2}$$

is the fraction of total excess demand variance that is attributable to relative disturbances. For given varances of the real disturbances, σ_ε^2 and σ_v^2, the magnitude of the Phillips-type response diminishes with the variance of the monetary

[15] The output expression neglects any differences in the sizes of markets – that is, there are no remaining systematic effects on $y_t(z)$ that are associated with the z-index.

[16] Barro and Grossman (1975, ch. 7) contains a related discussion for a model that has separate labor and commodity markets, but which treats expectations in an ad hoc manner.

[17] In an overlapping-generations model with a retirement period, this case may be typical. In this sort of model working households would have a small fraction of total wealth, so that β_s would be small. Further, the retired households, with a relatively large fraction of total wealth, would have short time horizons, so that β_d would be large.

growth rate, σ_m^2.[18] That is, when the monetary growth rate is less predictable, individuals are more inclined to associate observed price fluctuations in their markets with (aggregate) monetary movements. In that case, the reaction of output to a given monetary disturbance, m_t, would be correspondingly smaller. This type of effect has been discussed previously by Lucas (1973, p. 330), and it can be appropriately called the Lucas-hypothesis on the Phillips curve slope.[19] Given the negative effect of σ_m^2 on the Phillips slope, there is a sense in which more variable monetary growth has a 'stabilizing' effect on output. However, since this process reflects a monetary clouding that lessens the extent to which observed prices are a signal about relative prices, it seems intuitive that this type of stabilization would not be desirable in a full sense. Section 3.1 of this paper confirms that intuition.

Third, this type of model does not yield real effects of monetary disturbances that persist beyond one period – that is, only the current value of m_t enters into eq. (15). Some elements that could result in persistent effects are (1) the recognition that aggregate information is attained only gradually over time, rather than fully with a one-period lag; (2) elements of capital accumulation that would allow current changes in stocks to have a continued effect into subsequent periods; (3) adjustment costs in the supply and demand functions. Lucas's (1975) paper contains aspects of the first two of these elements. However, my analysis in the present paper does not incorporate any of these effects.

Fourth, the manner in which the current real shifts affect output brings out the key aspect of the information structure of the model – namely, each aggregate shift, v_t^d or v_t^s, has the same effect as the corresponding relative shift, $\varepsilon_t^d(z)$ or $\varepsilon_t^s(z)$. This behavior derives from the underlying assumption that participants in market z cannot tell what fraction of the observed movement in $P_t(z)$ reflects a relative price shift rather than an absolute shift. The precise way in which individuals would like to discriminate between these two types of shifts will be brought out below in section 3.1. It can also be noted here that the existence of an effect of unperceived monetary expansion on output, as discussed above, depends entirely on the inability of market participants to distinguish immediately between relative and absolute price shifts.

2.3. Price distributions

Given the price solutions in eqs. (10) and (12), the model determines distributions of prices both across markets and over time. It is convenient to focus

[18] If monetary disturbances had differential effects across markets – either systematic or random – one would anticipate a positive association between $\sigma_m{}^2$ and $\sigma_\varepsilon{}^2$. However, if the movement in $\sigma_\varepsilon{}^2$ is much less than one-to-one with $\sigma_m{}^2$, the qualitative conclusion about the Phillips curve slope would remain valid.

[19] I am currently attempting to test this hypothesis for the United States over the period 1890 to 1973. Lucas (1973) has performed some related tests for a cross section of countries during the post-World War II period. Lucas's results support the hypothesis, but his main evidence seems to rest on two outlying Latin American cases.

the discussion of these distributions on the problem of predicting the future price in a (randomly-selected) market z', based on information currently possessed by participants in market z. That is, I focus on the gap between $P_{t+1}(z')$ and $EP_{t+1}|I_t(z)$.[20] This gap can be usefully broken down into three independent components,

$$P_{t+1}(z')-EP_{t+1}|I_t(z) \equiv [P_{t+1}(z')-P_{t+1}]+[P_{t+1}-EP_{t+1}|I_t] + [EP_{t+1}|I_t-EP_{t+1}|I_t(z)], \tag{16}$$

where I_t denotes full current information. The information set I_t includes separate observations of m_t and v_t, whereas $I_t(z)$ includes only the combination of m_t, v_t, and $\varepsilon_t(z)$ that is implicit in an observation of $P_t(z)$. It turns out that the three components in eq. (16) are independently, normally distributed with zero mean, so that the variance of each component fully specifies its distribution. I will refer to these variances as τ_1^2, σ^2, and τ_2^2, respectively, and will use the symbol V to denote the sum of the three variances.

The first component corresponds to the distribution of relative prices at a point in time. Eqs. (10) and (12) (updated by one period) imply

$$P_{t+1}(z')-P_{t+1} = (1/\beta)[\theta_1+\theta_2+(\beta/\alpha)(1-\theta_1-\theta_2)]\varepsilon_{t+1}(z'),$$

which has zero mean (conditioned on $I_t(z)$). Using the expression for $\theta_1+\theta_2$ in eq. (11), the variance of relative prices can then be determined as

$$\tau_1^2 \equiv E[P_{t+1}(z')-P_{t+1}]^2|I_t(z) = \frac{\sigma_\varepsilon^2[\sigma_A^2+(\beta/\alpha)\sigma_\varepsilon^2]^2}{\beta^2(\sigma_A^2+\sigma_\varepsilon^2)^2}. \tag{17}$$

Not surprisingly, a key determinant of the relative price variance is σ_ε^2, the variance of relative excess demand.[21] More interestingly, there is also an effect of σ_A^2. This effect is positive as long as $\alpha > \beta$ holds, as I have been assuming. Therefore, an increase in the variance of aggregate excess demand leads to an increase in the variance of relative prices.[22] The reasoning for this effect is as follows. When σ_A^2 increases, the responsiveness of excess demand to locally

[20] Recall that $EP_{t+1}(z') = EP_{t+1}$ for all z' in this model.

[21] However, the effect is not unambiguously positive. Two sufficient conditions for a positive effect are $\alpha < 3\beta$ or $\sigma_\varepsilon^2 < \sigma_A^2$.

[22] Vining (1974) has a preliminary, I believe inconclusive, discussion of some post-World War II United States evidence on this issue. Graham (1930, p. 175) discusses some observations from the German hyperinflation that appear to support this hypothesis. Cairnes (1873) discusses the general idea that changes in money (gold) would have short-run effects on the dispersion of relative prices. His emphasis is on the (nonproportional) manner in which new money enters different parts of the economy, and on differences in the responsiveness of supply and demand for various types of commodities. Mills (1927 pp. 252-69) calculates measures of price dispersion for the United States from 1891 to 1926.

observed prices diminishes, because individuals are less inclined to associate price movements with shifts in relative excess demand. Accordingly, a given-size relative disturbance, $\varepsilon_t(z)$, requires a larger price movement in order to achieve clearing of the local market. This accentuated response of $P_t(z)$ to $\varepsilon_t(z)$ implies the increase in relative price variance, τ_1^2.

The second component of eq. (16) is the future price net of the price that is predictable based on full current information, I_t. Eqs. (12) and (13) imply

$$P_{t+1} - EP_{t+1}|I_t = (1/\beta)[\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2)](\beta m_{t+1} + v_{t+1}).$$

Note that this component has zero mean (conditioned on $I_t(z)$) and is independent of the first component. The variance of the future absolute price level can then be calculated as

$$\begin{aligned}\sigma^2 &\equiv E[P_{t+1} - EP_{t+1}|I_t]^2|I_t(z) \\ &= \frac{\sigma_A^2[\sigma_A^2 + (\beta/\alpha)\sigma_\varepsilon^2]^2}{\beta^2(\sigma_A^2+\sigma_\varepsilon^2)^2}.\end{aligned} \tag{18}$$

The effect of σ_A^2 on σ^2 is unambiguously positive if $\alpha > \beta$. The effect of σ_ε^2 is negative when $\alpha > \beta$ holds.

Finally, the third component in eq. (16) involves the distribution of relative information in terms of its implications for EP_{t+1}. Eqs. (12) and (13) imply

$$EP_{t+1}|I_t - EP_{t+1}|I_t(z) = (1/\beta)[(1-\theta_1-\theta_2)(\beta m_t + v_t) - (\theta_1+\theta_2)\varepsilon_t(z)].$$

It can be verified that this expression has zero mean conditioned on $I_t(z)$. This component is also independent of the first two components. The variance of relative information can then be calculated as

$$\begin{aligned}\tau_2^2 &= E[EP_{t+1}|I_t - EP_{t+1}|I_t(z)]^2|I_t(z) \\ &= \frac{\sigma_A^2\sigma_\varepsilon^2}{\beta^2(\sigma_A^2+\sigma_\varepsilon^2)}.\end{aligned} \tag{19}$$

This variance is increasing in both σ_A^2 and σ_ε^2.

The full variance of $P_{t+1}(z')$ about $EP_{t+1}|I_t(z)$ is the sum of the three component variances,

$$\begin{aligned}V &\equiv E[P_{t+1}(z') - EP_{t+1}|I_t(z)]^2|I_t(z) \\ &= \tau_1^2 + \sigma^2 + \tau_2^2 \\ &= \frac{1}{\beta^2(\sigma_A^2+\sigma_\varepsilon^2)}\left\{\left[\sigma_A^2 + (\beta/\alpha)\sigma_\varepsilon^2\right]^2 + \sigma_A^2\sigma_\varepsilon^2\right\}.\end{aligned} \tag{20}$$

It can be shown by straightforward differentiation that V is unambiguously

increasing in σ_ε^2,[23] and is unambiguously increasing in σ_A^2 as long as $\alpha > \beta$.

One aspect of the next part of the paper is an analysis of the impact of monetary policy on the predictability of future prices, as measured inversely by V in eq. (20). That analysis would be more meaningful if price predictability played some direct role in the commodity supply and demand functions – perhaps by affecting the costs of long-term nominal contracting. However, the present treatment does not incorporate this type of effect.

3. Monetary policy

Following Sargent and Wallace (1975), I now consider the role of monetary policy in a rational expectations framework. Monetary policy is identified here with a stochastic control rule for determining the time path of the money stock. My procedure differs from that of Sargent and Wallace in two major respects: first, the criterion for evaluating policy is different; and second, my analysis incorporates the dependence of certain coefficients of the model – in particular, the Phillips curve slope – on the underlying distributions of the excess demand shifts. Sargent and Wallace (1975, p. 5) evaluate policy by using a loss function that gives credit to stabilizing a measure of aggregate output, where this output measure is an aggregate analogue to my eq. (15). Their model corresponds in essential respects to dealing with the (geometric) average of the $y_t(z)$'s across the markets, where this averaging of eq. (15) over the n markets leads to an aggregate output expression in which the relative excess demand shifts, $\varepsilon_t^s(z)$ and $\varepsilon_t^d(z)$, do not appear. Stabilizing this measure of aggregate output would amount to giving no credit to changes in the composition of output that were responses to changes in relative supply and demand – that is, to changes in the composition of tastes, technology, etc. It seems clear that a loss function based on this simple measure of aggregate output would not be appropriate.

My earlier discussion of the output expression in eq. (15) stressed that the key aspect of the partial information structure of the model is the confusion between aggregate and relative shifts. It is possible to determine the output that would arise in each market if all participants were able to discriminate perfectly between these shifts – that is, under full current information where $I_t(z)$ includes observations on P_t and M_t. The output level under full current information (subsequently called full information output) can be compared with the output level determined in eq. (15). My proposed criterion for monetary policy is to minimize the expected squared gap between actual and full information output in each market.[24]

[23] That is, the positive effect on τ_2^2 in eq. (19) and the ambiguous (though likely positive) effect on τ_1^2 in eq. (17) dominate over the negative effect on σ^2 in eq. (18).

[24] My basic idea for this measure is that it should serve as an approximation to the expected loss of consumer surplus. Ideally, the criterion would be based directly on the behavior of individual expected utilities. Unfortunately, the present model is not set up to proceed in that fashion.

3.1. Prices and outputs under full current information

This model coincides with the one developed in part 2 except that P_t and M_t (and, hence, m_t, v_t and $\varepsilon_t(z)$) are now included in $I_t(z)$. The analysis proceeds as in section 2.2 until the derivation of $E P_{t+1}|I_t(z)$. Given the new information assumption, eq. (8) is now replaced by the simpler expression,

$$E P_{t+1}|I_t(z) = \Pi_1(M_{t-1}+m_t)+\Pi_5 u_t.$$

The remainder of the analysis follows the form of section 2.2. Using an asterisk to denote the full (current) information situation, the price level in market z turns out to be

$$P_t^*(z) = M_{t-1}+m_t+(1/\beta)(u_{t-1}+v_t)+(1/\alpha)\varepsilon_t(z). \tag{21}$$

It is convenient to rewrite here the price that arises under partial information, from eq. (10),

$$P_t(z) = M_{t-1}+[\theta_1+\theta_2+(\beta/\alpha)(1-\theta_1-\theta_2)][m_t+(1/\beta)(v_t+\varepsilon_t(z))] + (1/\beta)u_{t-1}.$$

In contrast with its effect on $P_t(z)$, the unanticipated change in the money stock, m_t, has a one-to-one effect on $P_t^*(z)$. Further, the current real disturbances, v_t and $\varepsilon_t(z)$, have different effects on $P_t^*(z)$. In particular, if $\alpha > \beta$, the response of $P_t^*(z)$ to the aggregate disturbance, v_t, is larger than that to the relative disturbance, $\varepsilon_t(z)$.

The result for full information output is

$$y_t^*(z) = (1/\alpha)(\alpha_s - H/\beta)v_t^d+(1/\alpha)(\alpha_d+H/\beta)v_t^s+(\alpha_s/\alpha)\varepsilon_t^d(z) + (\alpha_d/\alpha)\varepsilon_t^s(z)+(\beta_d/\beta)u_{t-1}^d+(\beta_s/\beta)u_{t-1}^s, \tag{22}$$

where $H \equiv \alpha_s\beta_d-\alpha_d\beta_s$. Again, it is convenient to rewrite the partial information result, this time from eq. (15),

$$y_t(z) = (H/\alpha)(1-\theta_1-\theta_2)m_t+(1/\alpha)[\alpha_s-(H/\beta)(\theta_1+\theta_2)] \times [v_t^d+\varepsilon_t^d(z)]+(1/\alpha)[\alpha_d+(H/\beta)(\theta_1+\theta_2)][v_t^s+\varepsilon_t^s(z)] +(\beta_d/\beta)u_{t-1}^d+(\beta_s/\beta)u_{t-1}^s.$$

There are several interesting contrasts between the results for $y_t(z)$ and those for $y_t^*(z)$. First, m_t has no effect on $y_t^*(z)$, which corresponds to the one-to-one effect of m_t on $P_t^*(z)$. Second, each aggregate shift, v_t^d or v_t^s, has a different

effect on $y_t^*(z)$ from the corresponding relative shift, $\varepsilon_t^d(z)$ or $\varepsilon_t^s(z)$. From inspection of the $y_t(z)$ and $y_t^*(z)$ expressions, it is clear that the two responses to the v_t's would coincide if $\theta_1+\theta_2 = 1$. This result obtains because $\theta_1+\theta_2 = 1$ signifies $\sigma_\varepsilon^2 = 0$, so that the aggregate – relative confusion cannot arise, and all aggregate shifts induce the appropriate output response. Note that the effect of m_t on $y_t(z)$ is zero in this case. It also follows that the response of $y_t(z)$ and $y_t^*(z)$ to the ε_t's would coincide if $\theta_1+\theta_2 = 0$, since $\sigma_A^2 = 0$ in that case. When $\theta_1+\theta_2$ is between zero and one, there will be divergences between the responses of $y_t(z)$ and $y_t^*(z)$ to m_t, v_t and $\varepsilon_t(z)$. This observation can be seen more easily by writing out an expression for the gap between actual and full information output,

$$y_t(z)-y_t^*(z) = (H/\alpha\beta)[(1-\theta_1-\theta_2)(\beta m_t+v_t)-(\theta_1+\theta_2)\varepsilon_t(z)]. \qquad (23)$$

If $H > 0$ and $0 < \theta_1+\theta_2 < 1$, it is clear from eq. (23) that (in relation to the full information situation) $y_t(z)$ reacts too much to the aggregate shifts, m_t and v_t, and not enough to the relative shifts, $\varepsilon_t(z)$. Eq. (23) also indicates again that the important informational division is between aggregate shifts, βm_t+v_t, and relative shifts, $\varepsilon_t(z)$. In a more general model it would also become relevant to separate the βm_t part of the aggregate shifts from the v_t part – or, put another way, to separate the monetary shift, m_t, from the real shift, $v_t+\varepsilon_t(z)$. For example, this other type of informational division would arise if u_t were no longer generated by a random walk. Appendix 1 deals with this case and clarifies some aspects of the two types of information divisions: aggregate versus relative and monetary versus real.

The proposed criterion for monetary policy is to minimize the expected squared gap between $y_t(z)$ and $y_t^*(z)$, which is denoted by Ω. Substituting for $\theta_1+\theta_2$ from eq. (11) and using eq. (23), the result is

$$\Omega \equiv \mathrm{E}[y_t(z)-y_t^*(z)]^2 | I_t(z)$$
$$= \frac{H^2\sigma_A^2\sigma_\varepsilon^2}{(\alpha\beta)^2(\sigma_A^2+\sigma_\varepsilon^2)}. \qquad (24)$$

This expression for the variance of output about its full information position will be used in the subsequent discussion of monetary policy.

3.2. *The optimal money variance*

Before introducing the possibility of monetary policy through feedback control on observed values of prices, outputs, etc., I consider here the role played by pure variance of money – that is, by σ_m^2. First, it is clear from eq. (23) that, if all the coefficients including $\theta_1+\theta_2$ were fixed, then an increase in σ_m^2 would lead to an increased variance of $y_t(z)$ about $y_t^*(z)$. Accordingly, in the

Sargent and Wallace model, where all coefficients are fixed, it is trivial that $\sigma_m^2 = 0$ would be optimal (and, hence, they do not discuss this issue). On the other hand, in my model an increase in σ_m^2 has effects that operate through the θ-coefficients. Specifically, the coefficient on the aggregate disturbance term in eq. (23) is

$$1-\theta_1-\theta_2 = \frac{\sigma_\varepsilon^2}{\beta^2\sigma_m^2+\sigma_v^2+\sigma_\varepsilon^2},$$

and this coefficient declines with σ_m^2. As σ_m^2 increases, individuals attribute a larger fraction of observed price movements to aggregate shocks and are, therefore, fooled less – in terms of the departure of $y_t(z)$ from $y_t^*(z)$ – for a given value of the aggregate shock, $\beta m_t + v_t$. In fact, as $\sigma_m^2 \to \infty$, the contribution of the aggregate disturbance term in eq. (23) to the output variance Ω, as calculated in eq. (24), approaches zero.[25] However, the contribution to Ω of the relative disturbance term in eq. (23), $(\theta_1+\theta_2)\varepsilon_t(z)$, is an increasing function of σ_m^2, so that the overall effect on Ω depends on two offsetting forces.

The nature of the net effect is apparent from eq. (24). The form of this expression implies that Ω is an increasing function of both σ_A^2 (which equals $\beta^2\sigma_m^2+\sigma_v^2$) and σ_ε^2. Hence, it is true in this model that the variance of output about its full information position is minimized by setting $\sigma_m^2 = 0$.[26] The reason for this result is that the policy criterion dictates getting output as close as possible to its full information position. An increase in any of the underlying variances, σ_m^2, σ_v^2, or σ_ε^2, clouds the picture, in the sense of making current price information a less accurate signal for market participants, and therefore makes it more difficult for individuals to get output close to full information output. To the extent that the variance of money, σ_m^2, can be controlled,[27] the smallest possible value would be optimal.[28]

The conclusion that $\sigma_m^2 = 0$ is optimal is basically in the spirit of the constant growth rate rule that has been advocated particularly by Friedman (for example, in Friedman, (1960, chapter 4)), and earlier by Simons (1948, pp. 181-?).

[25] Because $(1-\theta_1-\theta_2)^2$ approaches zero faster than $\sigma_m{}^2$ approaches infinity.

[26] In an earlier version of this paper I obtained the result that $\sigma_m{}^2 = 0$ would minimize Ω only under some configurations of the underlying parameters, and that $\sigma_m{}^2 = \infty$ was optimal in some other cases. Those conclusions depended on a misspecification in which EP_t, rather than EP_{t+1}, entered into the supply and demand functions. Another way to end up with $\sigma_m{}^2 = \infty$ as an answer is to change the objective function to the minimization of the variance of 'aggregate' output y_t about $Ey_t|I_{t-1}$ (essentially the Sargent and Wallace criterion), where the aggregation eliminates the $\varepsilon_t(z)$ terms from the output expression of eq. (15). This objective would definitely call for $\sigma_m{}^2 = \infty$ if aggregate real shifts were absent; that is, if $u_t^d \equiv u_t^s \equiv 0$. In the case where aggregate real shifts are present, the criterion would typically lead to a positive, but finite, value for $\sigma_m{}^2$.

[27] If there are significant money-control-type costs associated with reducing $\sigma_m{}^2$, then these costs would have to be weighed against the benefits from a lower variance. This sort of trade-off would lead to the choice of a positive value for $\sigma_m{}^2$.

[28] This result remains valid when the u_t process is no longer a random walk. See appendix 1.

The present result indicates that monetary policy is best when it is most predictable. In particular, an increase in money variance is nonneutral and leads to an increased variance of output about its full information position because money variance clouds the real picture and reduces the value of observed prices as allocative signals.

It is also useful to note here that the predictability of future prices is maximized by setting $\sigma_m^2 = 0$. That is, the variance V of $P_{t+1}(z')$ about $EP_{t+1}|I_t(z)$, which is indicated in eq. (20), is an increasing function of σ_A^2 (and, hence, of σ_m^2) as long as $\alpha > \beta$ holds. Therefore, the introduction of a price variance criterion into the objective function would not alter the above result.

3.3. Monetary policy as feedback control

I now consider the implications of complicating the money supply rule from the simple form of eq. (4) to include feedback effects from observed economic variables. This extension would allow the monetary authority to perform the countercyclical function of increasing money more rapidly when output is relatively low or prices are relatively high, and expanding money less rapidly in the reverse situations. The implications of this sort of monetary behavior depend crucially on the information set that is available to the monetary authority. There are two cases that have sharply divergent implications. In the first case, the monetary authority does not have more information than any of the market participants. Formally, the authority's current information set is I_{t-1}, which includes all relevant information with a one-period lag, but does not include an observation on P_t.[29] In this situation monetary policy can react (say, countercyclically) only to economic variables that have already been perceived by market participants. In the second case, the monetary authority has superior information about (some) current economic variables. In an extreme case the authority's information set would be I_t, which includes an observation on P_t. In this case the authority's feedback rule for ΔM_t can include some economic variables, such as aggregate values of current prices and outputs, which are not yet fully perceived by market participants. Not surprisingly, it turns out that countercyclical policy can be more potent under the second case than under the first (and, further, that policy may have zero potency under the first case). Finally, I assume in both cases that the market participants and the monetary authority have the same information about the form of the monetary rule. That is, the form of the rule is, itself, assumed to be a part of the information set I_{t-1} (and, hence, of $I_t(z)$). I consider in a later section some implications of differential information about the form of the monetary rule.

[29]In this situation the monetary authority actually has less information than any of the market participants since each individual has a current price observation, $P_t(z)$, in his information set, $I_t(z)$.

3.3.1. The monetary authority lacks superior information about the economy. I consider first the situation where the monetary authority's information set is I_{t-1}. The feedback control problem can be illustrated in this case by prescribing a monetary rule of the form,

$$\Delta M_t = m_t - \gamma v_{t-1}, \tag{25}$$

where, as before, $m_t \sim N(0, \sigma_m^2)$. Since v_{t-1} is last period's real shift to aggregate excess demand, the rule described by eq. (25) amounts to a countercyclical reaction to (one determinant of) last period's absolute price level if $\gamma > 0$. The form of the rule could be complicated to include a separate reaction to last period's 'aggregate' output or money stock, which would amount to introducing m_{t-1} and separate terms for v_{t-1}^d and v_{t-1}^s into eq. (25). The rule could also be extended to incorporate observations from period $t-2$ or earlier periods. However, these complications to the form of eq. (25) turn out to yield no additional insights, as should become clear from the subsequent discussion.[30]

When money is generated by the rule in eq. (25), the model can be solved out for prices and outputs using the same type of procedure as in section 2.2.[31] Since the formal procedure involves no important new elements, I will confine attention here to a presentation and discussion of the results. The solutions for $P_t(z)$, $EP_{t+1}|I_t(z)$, and $E\Delta M_{t+1}|I_t(z)$ turn out to be

$$\begin{aligned} P_t(z) &= M_{t-1} + [\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2) - \gamma\beta\theta_2] \\ &\quad \times [m_t + (1/\beta)(v_t + \varepsilon_t(z))] + (1/\beta)u_{t-1} - \gamma v_{t-1}, \\ EP_{t+1}|I_t(z) &= M_{t-1} + (\theta_1 + \theta_2 - \gamma\beta\theta_2)[m_t + (1/\beta)(v_t + \varepsilon_t(z))] \\ &\quad + (1/\beta)u_{t-1} - \gamma v_{t-1}, \\ E\Delta M_{t+1}|I_t(z) &= -\gamma\beta\theta_2[m_t + (1/\beta)(v_t + \varepsilon_t(z))]. \end{aligned}$$

In contrast to the earlier case in which there were no feedbacks to money (eqs. (4), (10), and (13), and $E\Delta M_{t+1}|I_t(z) = 0$), the new elements concern the γ-terms. These terms are of two types: those pertaining to v_{t-1} and those associated with v_t. First, v_{t-1} is contained in the information set, $I_t(z)$. Hence, the negative effect of v_{t-1} on M_t, as implied by eq. (25) if $\gamma > 0$, is fully perceived. As is generally the case for the perceived part of M_t, $P_t(z)$ moves in proportion to money – that is, the $-\gamma v_{t-1}$ term appears in the $P_t(z)$ expression. Since eq. (25) implies that the effect of v_{t-1} on M_t would also carry over to

[30] I have not discussed the possibility of monetary reaction to the array of $\varepsilon_{t-1}(z)$. Since the monetary authority is assumed to possess only the aggregate instrument, ΔM_t, one would not expect the pattern of relative excess demands to be an important input into policy decisions. In any event introducing the array of $\varepsilon_{t-1}(z)$ into eq. (25) would not change the basic results.

[31] The form of the $P_t(z)$ solution in eq. (6) would now include the additional term, $\Pi_6 v_{t-1}$.

M_{t+1}, it follows that $E P_{t+1}|I_t(z)$ also moves in accordance with $-\gamma v_{t-1}$. Therefore, the negative reaction of ΔM_t to v_{t-1} does not produce any gaps either between $P_t(z)$ and $E P_{t+1}|I_t(z)$ or between $M_t + E\Delta M_{t+1}|I_t(z)$ and $E P_{t+1}|I_t(z)$.

A second type of effect arises because the current excess demand shift, v_t, will have an effect next period on ΔM_{t+1}. Since v_t is not contained in $I_t(z)$, market participants form an estimate of the feedback effect on next period's money based on the expectation $\gamma E v_t|I_t(z) = \gamma\beta\theta_2[m_t + (1/\beta)(v_t + \varepsilon_t(z))]$. This term appears in the above expressions for $E\Delta M_{t+1}|I_t(z)$, $E P_{t+1}|I_t(z)$, and $P_t(z)$. Again, the response of ΔM_{t+1} to v_t does not produce any gaps either between $P_t(z)$ and $E P_{t+1}|I_t(z)$[32] or between $M_t + E\Delta M_{t+1}|I_t(z)$ and $E P_{t+1}|I_t(z)$.

The two feedback channels alter neither $P_t(z)$ relative to $E P_{t+1}$ nor the wealth term, $M_t + E\Delta M_{t+1} - E P_{t+1}$. It follows that there will be no effect on commodity supply and demand, as given in eqs. (1) and (2), and therefore no effect on output, $y_t(z)$. Because the market participants know the form of the money rule, and take this behavior into account in forming expectations of future prices and monetary growth rates, the feedback from v_{t-1} to M_t has no effect on the entire distribution of output.[33] The level of output continues to be determined by eq. (15). It follows trivially that the choice of the feedback parameter, γ, is irrelevant to the determination of the variance of output about its full information position, as determined in eq. (24).[34]

3.3.2. The monetary authority possesses superior information about the economy. The conclusions on the output effect of feedback control are radically different when the monetary authority has superior information that can be included in the money rule. The situation can be illustrated in the case where the authority has the information set I_t, which includes an observation of v_t.[35] In this case a

[32]Note that the supply and demand functions in eqs. (1) and (2) depend only on the expected real value of next period's money, $M_t + E\Delta M_{t+1} - E P_{t+1}$, which accords with the role of money as a store of value in this model. The current real money stock in market z, $M_t - P_t(z)$, might also enter these functions if the model incorporated the role of money as a mechanism for economizing on transaction costs (or if current real balances were simply included as a direct argument of individual utility functions). In that case $P_t(z)$ would not respond as much as $E P_{t+1}|I_t(z)$ to the expected movement in ΔM_{t+1}. The implied gap between $P_t(z)$ and $E P_{t+1}$ would then lead to effects on output, though the effects on actual and full information output would coincide. This sort of effect is analogous to the effect of systematic money growth on actual and full information output, as discussed in appendix 2.

[33]This type of result was first presented by Sargent and Wallace (1975, section 4) Their assumptions about the monetary authority's information set are analogous to those that I make in this section.

[34]Generally, there will be a nonzero effect of changes in γ on the predictability of future prices. The main effect is the following. When γ is high, the effect of v_{t+1} on P_{t+1} is attenuated because of the offsetting feedback effect on ΔM_{t+2}. Hence (at least if γ is not too large) P_{t+1} can be made more predictable, based on $I_t(z)$, by setting a positive value of γ.

[35]I do not consider here the possibility of superior information about the configuration of the ε_t's. Since the policymaker is assumed to possess only the aggregate instrument, ΔM_t, this sort of information would, in any case, be of only second-order use. Further, it seems much less plausible that the policymaker would actually have superior information about the relative shifts.

possible form of the monetary rule is

$$\Delta M_t = m_t - \delta v_t. \tag{26}$$

Again, $\delta > 0$ describes a countercyclical monetary policy, but this time the response of ΔM_t is to v_t, which is not a part of the individual information sets, $I_t(z)$. Again, the solution for prices and outputs can be derived from a procedure of the type discussed in section 2.2. The price results are now[36]

$$\begin{aligned} P_t(z) = {} & M_{t-1} + [\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2)] \\ & \times \{m_t + (1/\beta)[(1-\beta\delta)v_t + \varepsilon_t(z)]\} + (1/\beta)u_{t-1}, \end{aligned} \tag{27}$$

$$\begin{aligned} \mathrm{E}P_{t+1}|I_t(z) = {} & M_{t-1} + (\theta_1+\theta_2)\{m_t + (1/\beta)[(1-\beta\delta)v_t + \varepsilon_t(z)]\} \\ & + (1/\beta)u_{t-1}. \end{aligned}$$

The feedback from v_t to ΔM_t implies that $\beta m_t + (1-\beta\delta)v_t$ is now the aggregate excess demand shift that affects $P_t(z)$ and $\mathrm{E}P_{t+1}|I_t(z)$. Therefore, the variance of aggregate demand is now

$$\sigma_A^2 = \beta^2\sigma_m^2 + (1-\beta\delta)^2\sigma_v^2.$$

It is apparent that raising the feedback parameter, δ, will reduce σ_A^2 as long as $\delta < 1/\beta$ applies. Further, setting $\delta = 1/\beta$ would minimize σ_A^2 for a given value of σ_m^2. (The combination of $\delta = 1/\beta$ and $\sigma_m^2 = 0$ would yield $\sigma_A^2 = 0$.)

The formula for Ω, the variance of output about its full information position,[37] is still that given in eq. (24), and the formula for V, the variance of the absolute future price level, is still that given in eq. (20). In particular, reductions in σ_A^2 unambiguously reduce Ω, and such reductions also unambiguously reduce V if $\alpha > \beta$ applies. It is then clear that $\delta = 1/\beta$ (along with $\sigma_m^2 = 0$) yields the optimal money rule of the form of eq. (26). This parameter choice implies that the aggregate excess demand shift is $m_t + (1-\beta\delta)v_t \equiv 0$. That is, there would be sufficient feedback from v_t to ΔM_t so that the direct effect of v_t on excess demand would be fully offset by an inverse movement of ΔM_t.[38]

Although the above form of stabilization policy seems obvious under the

[36]Given eq. (26), it follows that $\mathrm{E}\Delta M_{t+1}|I_t(z) = 0$.

[37]The formula for $y_t(z)$ from eq. (15) is modified only in the coefficients of v_t^d and v_t^s. The new terms are

$$(1/\alpha)\{\alpha_s - (H/\beta)[(1-\beta\delta)(\theta_1+\theta_2) + \beta\delta]\}v_t^d,$$
$$(1/\alpha)\{\alpha_d + (H/\beta)[(1-\beta\delta)(\theta_1+\theta_2) + \beta\delta]\}v_t^s.$$

The formula for $y_t(z) - y_t^*(z)$ in eq. (23) is modified only by replacing v_t with $(1-\beta\delta)v_t$. However, the $\theta_1+\theta_2$ coefficient that appears in this expression now involves $(1-\beta\delta)^2\sigma_v^2$, rather than σ_v^2 (see eq. (11)).

[38]This result can be generalized to a case where the policymaker has only partial information about current variables. As long as the monetary authority possesses some information that is not possessed by all market participants, there would be some potential role for countercyclical adjustments in money.

assumed superiority in the monetary authority's information set, the way that it works is somewhat subtle. In particular, the stabilization policy does not operate to eliminate any output effects of shifts in v_t^d or v_t^s, but, rather, it works by removing discrepancies between the movements of actual and full information output. Assume, for example, that there is no monetary feedback ($\delta = 0$), and that v_t^d is positive while v_t^s is zero. According to eq. (27), this unanticipated aggregate demand shift would affect prices in all markets equiproportionately – that is, there would be a shift in the absolute price level, but no shift in relative prices. However, participants in market z would not be able fully to distinguish this shift from a relative price change, and, therefore, the movement in output would depart from the movement in full information output. In the case where $H > 0$, eq. (23) indicates that actual output would increase too much in this situation. Suppose, now, that a stabilization policy is adopted that implies a negative response of ΔM_t to v_t so that the net disturbance, $(1-\beta\delta)v_t$, is maintained at zero. Eq. (27) indicates that neither relative nor absolute prices would then be affected by the positive value of v_t^d. In that case there is no possibility of a confusion between absolute and relative shifts, and the movement in v_t^d cannot lead to a departure of actual from full information output. Further, as is clear from eq. (22), the movement in ΔM_t itself does not affect the response of full information output. Therefore, in this example, both actual and full information output would increase with the positive value of v_t^d in accordance with the coefficient shown in eq. (22).

Since the stabilization policy works by preventing a confusion between absolute and relative price changes, it is also clear that an alternative to the active stabilization policy would be the provision of the information about current economic variables. If the monetary authority actually had more rapid observations of v_t, they could convey this information to the public. This information would then augment the information set, $I_t(z)$, that is used to form expectations about P_{t+1}. Once v_t is observable, it is clear that shifts in v_t can no longer lead to confusions between relative and absolute price changes. Hence, as in the case of the countercyclical monetary policy described above, movements in v_t would not produce discrepancies between actual and full information output. In other words, when the monetary authority has superior information about the economy, the provision of the information to the public is an alternative to an active stabilization policy. An argument for the superiority of stabilization policy would have to be based on the costs of transmitting and using the relevant information.[39] In particular, it could be argued that the

[39]The active stabilization policy and the information-provision policy do have different implications for the predictability of future prices. The information-provision policy ($\delta = 0$) in eq. (26), but with v_t contained in $I_t(z)$, would involve a higher variance of future prices. Essentially, the movements in v_t and the associated movements in P_t would be perceived currently, but these movements would still not be predictable at date $t-1$. On the other hand, the stabilization policy described above completely eliminates fluctuations in P_t associated with movements in v_t, and therefore makes P_t more predictable at date $t-1$.

existence of an active stabilization policy motivates individuals to reduce expenditures that are aimed at augmenting their information sets. If there are economies-of-scale in information production, there could be a net social gain along these lines.[40]

3.3.3. The monetary authority has superior information about the monetary rule. Taylor (1975) has stressed the idea that individuals would not have perfect information about the form of the monetary rule. In this situation it is plausible that the monetary authority would have better information than the general public about its own future actions. In fact, this situation seems most plausible when the policymaker lacks a consistent objective function, such as the minimization of the variance of output about its full information position.[41] In any case, if the monetary authority has better information about its monetary rule, there is the possibility of controlling money so as to systematically fool the public. Taylor has pointed out that this sort of deception can be carried out during 'transition' periods during which individuals are modifying their beliefs (along Bayesian adaptation lines) about the form of policy.[42] In Taylor's model there is also an optimal, nonzero amount of this deception. His model appears to support this type of action because the policymaker's objective function does not reflect individual preferences.[43] In my model, there appears to be no basis for policy deception as long as the policymaker's objective is based on minimizing the gap between actual and full information output.[44]

A simple form of policy deception arises when individuals believe that $\Delta M_t = m_t$, where $m_t \sim N(0, \sigma_m^2)$, but where the monetary authority knows (determines) that m_t is generated by a distribution other than $N(0, \sigma_m^2)$. Consider, for example, the case where m_t can be set by the monetary authority at any desired level, while, in the 'short run', holding fixed people's belief that m_t has zero mean and variance σ_m^2.[45] In this case it is clear that the choice of m_t has a systematic

[40]Of course, once this sort of information externality is introduced, it is also natural to consider the negative externalities associated with governmental incentive and control.

[41]An unpredictable objective function would be one possible rationale for the existence of m_t, the stochastic part of money.

[42]Sargent and Wallace (1974, p. 16) argue that there is no way for the monetary authority to systematically fool the public, even in the short run.

[43]His objective function gives positive credit to reducing unemployment throughout the relevant range. The analogy to my model would be to credit expansions of output even when it was already above its full information position.

[44]The normative case for policy deception could be based on external effects, such as income taxation, unemployment compensation, etc., that are not incorporated into my model. This idea is discussed by Phelps (1972) and also by Hall (1973), who argues: '... the benefits of inflation derive from the use of expansionary policy to trick economic agents into behaving in socially preferable ways even though their behavior is not in their own interest ...'. Prescott (1975) downplays the importance of external effects in this context. In any case the possible external effects seem to have more pertinence for long-term allocative policies, such as the design of tax and welfare system, than for countercyclical monetary policy.

[45]The monetary authority might, instead, be reacting to v_{t-1} by a feedback rule of the form of eq. (25), but individuals currently believe that $\gamma = 0$.

effect on current output, as determined in eq. (15). However, it is also clear from eq. (23) that the expected squared gap between actual and full information output would be minimized by choosing $m_t = 0$.[46] Given the objective of minimizing gaps between actual and full information output, it is not surprising that optimal behavior rules out policy deception. In this sort of framework the best monetary policy is always the policy that is most predictable.[47] The obvious policy implication is for the monetary authority to make known in advance its intentions about money growth,[48] which is again the basic philosophy behind the constant growth rate rule.

4. Conclusions

I will conclude by highlighting some of the main results that deal with the effects of money variance and with the role of monetary policy. An increased variance of money makes it more difficult for individuals to react appropriately to the real shifts in the economy. There are two important types of responses to an increased money variance. First, since individuals react by attributing a larger fraction of observed price movements to monetary causes, there is a smaller effect of a given size monetary disturbance on output – that is, the magnitude of the Phillips curve slope is smaller. Second, the associated compounding of individual information problems leads both to a higher variance of output about its full (current) information position and to a reduced predictability of future prices. It also leads to an increase in the variance of relative prices across markets.

From the standpoint of monetary policy, it is clear that pure variance of money is harmful, essentially because it clouds the real picture for individuals. The analysis of monetary policy as feedback control is more complicated since the results hinge on the relative information positions of the monetary authority and the public. When the authority lacks superior information, the feedback to money must be based on economic variables that have already been perceived by the public. In this circumstance the choice of feedback control parameters has no implications for the entire distribution of output. On the other hand, if the monetary authority has superior information about the economy –

[46]If the monetary authority sets $m_t = 0$ continually, this action would also lead people to believe (along Bayesian lines) that $\sigma_m^2 = 0$. I have not dealt explicitly with the effects of fooling people about the value of σ_m^2. Presumably, the variance of actual about full information output is minimized when perceptions about σ_m^2 are correct.

[47]There is a sense in which this conclusion is violated for the case where the monetary authority has superior information about the economy. In particular, the feedback rule from v_t to ΔM_t described by eq. (26) would be ineffective if people fully perceived the countercyclical money response – δv_t, while still remaining in the dark about v_t. On the other hand, it is desirable even in this case for people to know the form of the monetary rule.

[48]More specifically, the Federal Reserve should publicize, as rapidly as possible, the proceedings of its Open Market Committee.

which, for some reason, it does not provide to the public directly – then the appropriate feedback response to the extra information can move output closer to its full information position.

Basically, the conclusions for monetary policy are in accord with the philosophy behind Friedman's proposal for a constant growth rate rule. It is only to the extent that the monetary authority has superior economic information (as well as the appropriate objectives), and to the extent that providing information to the public is costly, that there is a call for departures from the constant growth rate rule. Further, if the attempt to use countercyclical policy to exploit the superior information results in a higher variance of money, σ_m^2, there would be a tradeoff between the beneficial effects from the countercyclical elements (the negative correlation between v_t and ΔM_t) and the adverse effects from pure monetary variance.[49]

It may be useful to discuss the role of feedback control of money in the case of a concrete example.[50] The United States economy in 1974 was affected by two important real shocks: the oil cartel and the shortfall of agricultural harvests. Although my model has been constructed to deal with a closed economy, it seems that either of these shocks can be represented by a downward movement in aggregate real supply, v_t^s, and a lesser downward movement in aggregate real demand, v_t^d. (I am abstracting here from effects on relative supply and demand, which would be quantitatively important for these shocks, but which would not affect the essential parts of my story.) It follows that output (in a 'typical' market which experiences zero relative shifts) would fall while prices would rise. What is the role for monetary policy in this situation? The present analysis suggests that there is a substantive role only to the extent that the monetary authority has better information than the public about the disturbances, or, possibly, about their implications for the economy. Perhaps the most obvious observation about the oil and agricultural shocks is the extent to which they are perceived. Hence, the approach in this paper argues that there is no role for monetary policy in offsetting these real shifts.[51,52] Adverse shifts like the oil and agricultural crises will reduce output and cause painful relative adjustments no matter what the reaction of the monetary authority. Added monetary noise would only complicate and lengthen the process of adjustment.

[49]This type of tradeoff is discussed in Friedman (1953).

[50]Gordon (1974) discusses the same example. Perhaps not surprisingly, he reaches very different conclusions.

[51]Further, to the extent that there is any role it would be a contraction of ΔM_t in response to the positive value of $v_t \equiv v_t^d - v_t^s$.

[52]The present analysis implies that having the monetary authority announce that there had been an oil or agricultural crisis (or, perhaps, telling people that these crises meant lower output and higher prices) would be equivalent to the appropriate active response of money. In this case it seems that both the announcement and the active policy would have negligible effects. In fact, the announcement would be somewhat preferable since it would not involve the danger of introducing added variance into the money supply.

Appendix 1: Results when u_t is a first-order Markov Process

I consider here the case where the real excess demand shift, u_t, is generated by a first-order Markov Process,

$$u_t = \lambda u_{t-1} + v_t,$$

where

$$0 \leqq \lambda \leqq 1.$$

Under this specification the magnitude of λ determines the extent to which the current shift, v_t, persists in its effect on u_{t+1}, and, hence, on P_{t+1}. The case treated in the main text corresponds to $\lambda = 1$ – that is, to the situation where v_t affects u_{t+1} on a one-to-one basis.

The solution for prices and outputs can be derived in accordance with the general procedure developed in section 2.2 of the text. Since no new elements are involved, I will simply indicate the results. It is convenient to define the parameter,

$$\hat{\lambda} = \frac{\lambda}{\lambda + (\alpha/\beta)(1-\lambda)}.$$

If $\alpha > \beta$ then the $\hat{\lambda}$ parameter satisfies the conditions $0 \leqq \hat{\lambda} \leqq \lambda \leqq 1$, $\hat{\lambda} \to 0$ as $\lambda \to 0$, and $\hat{\lambda} \to 1$ as $\lambda \to 1$. Recalling that $H \equiv \alpha_s\beta_d - \alpha_d\beta_s$, the results for prices and outputs are now

$$\begin{aligned}
P_t(z) &= M_{t-1} + [\theta_1 + \hat{\lambda}\theta_2 + (\beta/\alpha)(1-\theta_1-\hat{\lambda}\theta_2)] \\
&\quad \times [m_t + (1/\beta)(v_t + \varepsilon_t(z))] + (\hat{\lambda}/\beta)u_{t-1}, \\
E P_{t+1} | I_t(z) &= M_{t-1} + (\theta_1 + \hat{\lambda}\theta_2)[m_t + (1/\beta)(v_t + \varepsilon_t(z))] \\
&\quad + (\lambda\hat{\lambda}/\beta)u_{t-1}, \\
y_t(z) &= (H/\alpha)(1-\theta_1-\hat{\lambda}\theta_2)m_t + (1/\alpha)[\alpha_s - (H/\beta)(\theta_1+\hat{\lambda}\theta_2)] \\
&\quad \times [v_t^d + \varepsilon_t^d(z)] + (1/\alpha)[\alpha_d + (H/\beta)(\theta_1+\hat{\lambda}\theta_2)][v_t^s + \varepsilon_t^s(z)] \\
&\quad + \lambda\left[\frac{\alpha_s(1-\lambda)+\beta_s\lambda}{\alpha(1-\lambda)+\beta\lambda}\right]u_{t-1}^d + \lambda\left[\frac{\alpha_d(1-\lambda)+\beta_d\lambda}{\alpha(1-\lambda)+\beta\lambda}\right]u_{t-1}^s, \\
y_t^*(z) &= (1/\alpha)[\alpha_s - (H/\beta)\hat{\lambda}]v_t^d + (1/\alpha)[\alpha_d + (H/\beta)\hat{\lambda}]v_t^s + (\alpha_s/\alpha)\varepsilon_t^d(z) \\
&\quad + (\alpha_d/\alpha)\varepsilon_t^s(z) + \lambda\left[\frac{\alpha_s(1-\lambda)+\beta_s\lambda}{\alpha(1-\lambda)+\beta\lambda}\right]u_{t-1}^d \\
&\quad + \lambda\left[\frac{\alpha_d(1-\lambda)+\beta_d\lambda}{\alpha(1-\lambda)+\beta\lambda}\right]u_{t-1}^s,
\end{aligned}$$

$$y_t(z)-y_t^*(z) = (H/\alpha\beta)[\beta m_t+\hat{\lambda}v_t-(\theta_1+\hat{\lambda}\theta_2)(\beta m_t+v_t+\varepsilon_t(z))],$$

$$\Omega \equiv \mathrm{E}[y_t(z)-y_t^*(z)]^2 \big| I_t(z)$$

$$= \frac{H^2}{(\alpha\beta)^2(\sigma_A^2+\sigma_\varepsilon^2)}\left[(1-\hat{\lambda})^2\beta^2\sigma_m^2\sigma_v^2+\beta^2\sigma_m^2\sigma_\varepsilon^2+(\hat{\lambda})^2\sigma_v^2\sigma_\varepsilon^2\right],$$

where $\sigma_A^2 \equiv \beta^2\sigma_m^2+\sigma_v^2$ in the last expression. The original results in section 2.2 of the text correspond, in each case, to $\lambda = \hat{\lambda} = 1$.

The principal new results from allowing $\hat{\lambda} \neq 1$ are brought out in the above expression for Ω. When $\hat{\lambda} = 1$, the aggregate real shift, v_t, is permanent, in the sense of affecting u_{t+1} on a one-to-one basis. More importantly, the monetary variable, M_t, and the aggregate real shift, u_t, are in this case generated by processes of the same form – that is, by random walks. In this situation the current disturbance associated with money, βm_t, and the current movement in the aggregate real disturbance, v_t, have identical implications for the future price level, P_{t+1}. Therefore, when $\hat{\lambda} = 1$, it is unnecessary for individuals who are interested in forecasting P_{t+1} to separate out the βm_t part of the current excess demand shift from the v_t part. The only concern is with separating the total aggregate shift, βm_t+v_t, from the relative shift, $\varepsilon_t(z)$ (since $\varepsilon_t(z)$ is purely transitory and has no impact on P_{t+1}). In the Ω-expression, $\hat{\lambda} = 1$ implies that the $\sigma_m^2\sigma_v^2$ interaction term vanishes, and the remaining terms can be combined into an interaction term between σ_ε^2 and $\beta^2\sigma_m^2+\sigma_v^2 \equiv \sigma_A^2$ (as in eq. (24)). In this case the problem of separating permanent from transitory shifts in order to forecast P_{t+1} amounts to separating aggregate from relative shifts.

The other polar case is $\hat{\lambda} = 0$, which corresponds to the v_t shifts being purely transitory. In that case v_t and $\varepsilon_t(z)$ are generated by processes of the same form and the $\sigma_v^2\sigma_\varepsilon^2$ interaction term vanishes from the Ω-expression. The remaining terms can then be combined into an interaction term involving $\beta^2\sigma_m^2$ and $\sigma_v^2+\sigma_\varepsilon^2$. That is, the separation between permanent and transitory reduces in this case to a separation between monetary and real.

In the general case where $\hat{\lambda}$ is in the interval between zero and one, all three interaction terms appear in the Ω-expression. Individuals would then be concerned with the full separation of the current excess demand shift, $\beta m_t+v_t+\varepsilon_t(z)$, into its three components. The separation between permanent and transitory would entail two types of divisions of current excess demand shifts: aggregate versus relative and monetary versus real.

From the standpoint of monetary policy, the important aspect of the extended model is that Ω is still a strictly increasing function of σ_m^2. (This property can be verified from straightforward differentiation of the Ω-expression.) Hence, the compounding of the information problem to include a separation of monetary versus real along with a separation of aggregate versus relative does not alter the conclusion that monetary noise makes the information problem more difficult.

Finally, it can be noted that the extension of the model to a first-order Markov process for u_t has been carried out within the framework where the monetary disturbance, m_t, and the relative shift, $\varepsilon_t(z)$, are purely white noise processes. It would be possible to introduce some serial dependence into these processes. However, it is already apparent from the above discussion of the u_t process that the crucial consideration is the relation between the processes that generate M_t, u_t and $\varepsilon_t(z)$. When these three processes assume different forms there will be an information problem associated with dividing currently observed excess demand shifts into its three components, The above case, in which the u_t-process is first-order Markov (with $\hat{\lambda} \neq 0$ or 1) and m_t and $\varepsilon_t(z)$ are white noise, is one way in which the processes for M_t, u_t, and $\varepsilon_t(z)$ can take on different forms. Further alteration of the m_t or $\varepsilon_t(z)$ processes would not seem to change the basic picture, at least in terms of the implications for money variance, σ_m^2.

Appendix 2: Systematic growth in money and output

This section extends the analysis of the text in two respects. First, the systematic growth rate of money, g in eq. (4), is allowed to be nonzero. Second, the growth rate of k_t, the systematic part of excess demand which is defined as $-\beta\rho$ in eq. (5), is allowed to be nonzero. It is clear that the introduction of these systematic growth elements (which are included in the information set, $I_t(z)$) would not affect the gap between actual and full information output. Therefore, the present discussion is limited to the effects of systematic growth on $P_t(z)$, $EP_{t+1}|I_t(z)$, and $y_t(z)$. The analysis here returns to the case where u_t is generated by a random walk ($\hat{\lambda} = 1$, in the terminology of appendix 1).

Formally, the extended model can be solved by the method of section 2.2 of the text if the solution form for $P_t(z)$ in eq. (6) is extended to include a time trend and a constant term – that is, $\Pi_6 t + \Pi_7$. Additional complications to the systematic parts of M_t and k_t – for example, to allow nonconstant growth rates – would be reflected as additional terms in the solution form for $P_t(z)$. Given the extended form of the solution, the procedure for solving the model is the same as that employed in section 2.2.

The solution for $P_t(z)$ coincides with eq. (10) except for the inclusion of systematic effects associated with g and ρ. The extended result is

$$P_t(z) = M_{t-1} + g + [\theta_1 + \theta_2 + (\beta/\alpha)(1-\theta_1-\theta_2)][m_t + (1/\beta)(v_t + \varepsilon_t(z))]$$
$$+ (1/\beta)u_{t-1} - \rho t + (\alpha/\beta)(g-\rho) + \rho.$$

Since $M_{t-1} + g$ is now the fully perceived part of M_t, this term has a one-to-one effect on $P_t(z)$. The $-\rho t$ term indicates that the systematic growth of k_t at rate $-\beta\rho$ would generate a systematic growth in the price level at rate $-\rho$ if M were constant. With M growing steadily at rate g, the net systematic

growth rate of the price level is $g-\rho$. This systematic rate of inflation appears additionally in the term $(\alpha/\beta)(g-\rho)$ as a positive effect on $P_t(z)$ for a given value of the nominal money stock. Equivalently, systematic inflation reduces the (expected) holding of real balances. I will discuss below the meaning of the final term, $+\rho$, in the $P_t(z)$ expression.

The expected price level for next period is now

$$E P_{t+1}|I_t(z) = M_{t-1}+2g+(\theta_1+\theta_2)[m_t+(1/\beta)(v_t+\varepsilon_t(z))]$$

$$+(1/\beta)u_{t-1}-\rho(t+1)+(\alpha/\beta)(g-\rho)+\rho.$$

In particular, the gap between $P_t(z)$ and $E P_{t+1}|I_t(z)$ now includes the term $-(g-\rho)$, which is the negative of the systematic rate of inflation. In this model, where commodity supply and demand depend on $P_t(z)-E P_{t+1}|I_t(z)$, it is this effect of systematic growth that leads to influences on output.

The solution for output is determined by substituting the price results into the commodity supply or demand function, as given in eqs. (1) and (2). The result for $y_t(z)$ coincides with eq. (15) except for the new systematic effects,

$$y_t(z) = (\beta_s/\beta)k_t^d+(\beta_d/\beta)k_t^s-(H/\beta)(g-\rho)+\ldots,$$

where the terms that appear in eq. (15) have been omitted. In determining output it is necessary to specify separately the systematic demand and supply, k_t^d and k_t^s, as well as the excess demand, k_t. I assume that individuals plan the systematic growth rate of real balances (real wealth) to equal the systematic growth rate of output. From the above expressions for $P_t(z)$ and $E P_{t+1}|I_t(z)$, it is clear that the systematic growth rate of real balances is ρ. From the $y_t(z)$ expression it is clear that the systematic growth rate of output depends on the systematic growth rates of k_t^d and k_t^s. The condition that the systematic growth of real balances coincides with the systematic growth of output therefore implies a condition on the time paths of k_t^d and k_t^s. Using also the condition that $k_t = -\beta\rho t$, and setting $k_0^d = k_0^s = 0$ for convenience, it can be determined that

$$k_t^d = \rho(1-\beta_s)t,$$

$$k_t^s = \rho(1+\beta_d)t.$$

Substitution into the above expression for $y_t(z)$ yields

$$y_t(z) = \rho t-(H/\beta)(g-\rho)+\ldots.$$

That is, the systematic growth rate of output is, indeed, equal to ρ.

The other property of the output expression is that an increase in the systematic inflation rate, $g-\rho$, reduces output (both actual and full information) if $H > 0$. The mechanism is as follows. An increase in $g-\rho$ implies a lower real rate of return on money, which is the only store of value in the model. Accordingly, there is a substitution effect that reduces current labor supply and raises current consumption demand.[53] This shift raises output when $H > 0$. Of course, this sort of effect is operative here because money serves as the only store of value. Further, the present analysis does not deal with any benefits of holding money that are associated with transaction costs. For these reasons, it seems that the present model is probably more useful for an analysis of unperceived monetary change than for an analysis of systematic inflation.

Finally, I can now comment on the presence of the $+\rho$ terms in the above price expressions. In eqs. (1) and (2), excess commodity demand depends on expected next period's real balances, $M_t + \mathrm{E}\Delta M_{t+1}|I_t(z) - \mathrm{E}P_{t+1}|I_t(z)$. This formulation is reasonable as long as desired real balances are constant. More generally, it is a gap between expected and desired real balances that would produce an effect on excess commodity demand. It is now apparent that (desired) real balances grow at rate ρ in this model. If the effects of real balances on excess commodity demand are adjusted to take account of this systematic growth in desired real balances, the $+\rho$ terms would no longer appear in the above price expressions. The expression for $y_t(z)$ would be unaffected by this adjustment. Of course, no adjustment at all is required for the case in the text where $\rho = 0$ was assumed.

[53]In a life-cycle model these effects would also alter the distribution of wealth by age. I have not dealt with this type of distribution effect.

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